BRITISH MATHEMATICAL OLYMPIAD

Round 2 : Wednesday, 23 February 2000

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.*

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (6-9 April). On the final morning of the training session, students sit a paper with just 3 Olympiad-style problems. The UK Team - six members plus one reserve - for this summer's International Mathematical Olympiad (to be held in South Korea, 13-24 July) will be chosen immediately thereafter. Those selected will be expected to participate in further correspondence work between April and July, and to attend a short residential session before leaving for South Korea.

Do not turn over until **told to do so.**

BRITISH MATHEMATICAL OLYMPIAD

1. Two intersecting circles C_1 and C_2 have a common tangent which touches C_1 at P and C_2 at Q. The two circles intersect at M and N, where N is nearer to PQ than M is. Prove that the triangles MNP and MNQ have equal areas.

2. Given that x, y, z are positive real numbers satisfying $xyz = 32$, find the minimum value of
$$x^2 + 4xy + 4y^2 + 2z^2.$$

3. Find positive integers a and b such that
$$(\sqrt[3]{a} + \sqrt[3]{b} - 1)^2 = 49 + 20\sqrt[3]{6}.$$

4. (a) Find a set A of ten positive integers such that no six distinct elements of A have a sum which is divisible by 6.

   (b) Is it possible to find such a set if "ten" is replaced by "eleven"?